Math 95. Section 4.1 (a): Systems of Linear Equations in Two Variables; Solving Systems by Graphing and Substitution. And Sec. 4.3 Applications.

1. A System of Linear Equations: a pair (or more) of linear eqns.

\[
\begin{align*}
  y &= 3x + 2 \\
  y &= -\frac{1}{2}x + 4
\end{align*}
\]

A. Solution: the point \((x, y)\) that fits both eqns.
The graph it is the point of intersection.

Ex. 1: Determine whether \((-3, 3)\) is a solution to the system:

\[
\begin{align*}
  3x - y &= -12 \\
  x - y &= 0
\end{align*}
\]

\[
\begin{align*}
  3(-3) - 3 &= -12 \\
  -9 - 3 &= -12 \checkmark \\
  -3 - 3 &= 0 \\
  -6 &\neq 0 \text{ Not a soln. to the system.}
\end{align*}
\]
B. Number of Possible Solutions:

1. Inconsistent: There is No solution.
   The lines are parallel.

2. Consistent: Is at least one solution:
   a) independent: One solution
   b) dependent: An infinite number of solutions.
   They represent the same line.
II. Methods of Solution:

1. Graphing: 
   a good check for all types of solutions and a good visual
   It is not always accurate.

2. Substitution: 
   Solve for one variable in one eqn.
   Substitute for that variable in the other eqns.

3. Elimination: 
   Put both eqns in standard form
   $Ax + By = C$. Use multiples to eliminate one
   of the variables.
II. Solving by Graphing: good for approximation, a good visual of the situation, also a good check for your symbolic solutions.

Ex. 1: \[ L_1: \begin{align*} x + y &= 4 \\ x - y &= 2 \end{align*} \]

\[ L_1: \begin{align*} y &= 4 - x \\ y &= -x + 4 \end{align*} \]

\[ L_2: \begin{align*} x - y &= 2 \\ -y &= -x + 2 \end{align*} \]

\[ y = 2 - x \]

\[ \text{Solve: } (3, 1) \]

\[ \text{Check: } 3 + 1 = 4 \checkmark \]

\[ 3 - 1 = 2 \checkmark \]
Ex. 2: \( L_1 \)
\( 2x + y = 1 \)
\( 4x + y = 4 \)

\( L_1: \quad y = -2x + 1 \)

\( L_2: \quad y = -4(x + 1) \)

\( \left(1 \frac{1}{2}, -2 \frac{1}{2}\right) \)
III. Substitution:
1. Choose one of the equations and solve it for one of the variables.
2. Substitute what you get into the other equation and solve.
3. Back solve and check your solution in both equations.

Use substitution to solve the following systems of linear equations

Ex. 3: \[ \begin{align*}
3x - 2y &= 4 \\
2x - y &= 5
\end{align*} \]

\[ \begin{align*}
y &= -2x + 1 \\
y &= 2x - 1
\end{align*} \]

\[ \begin{align*}
3x - 2(2x - 1) &= 4 \\
3x - 4x + 2 &= 4 \\
x + 2 &= 4
\end{align*} \]

\[ x = -2 \]

\[ y = 2(-2) - 1 \\
y = -5
\]
Ex. 4: 
\[
\begin{align*}
2x + y &= 4 \\
4x - y &= -1 \\

\Rightarrow y &= -2x + 4 \\

4x - (-2x + 4) &= 1 \\
yx + 2x - 4 &= 1 \\
x - 4 &= 1 \\
6x &= 3 \\
x &= \frac{1}{2} \\
y &= -2\left(\frac{1}{2}\right) + 4 \\
&= -1 + 4 \\
y &= 3 \\

\left(\frac{1}{2}, 3\right)
\end{align*}
\]
Ex. 5: \[
\begin{align*}
\begin{cases}
2x - y &= -4 \\
4x - 2y &= 6
\end{cases}
\end{align*}
\]
\[
\begin{align*}
-y &= -2x - 4 \\
y &= 2x + 4
\end{align*}
\]

L: \(y = 2x + 4\)

\[
-2x = -4x + b
\]

⇒: \(x = \frac{b}{2} \neq 3\)

\[
4x - 2(2x + 4) = 6
\]

Variable disappears and you get a false statement. The system is inconsistent, thus is No Solution. The lines are parallel!
III. Applications: Use substitution to solve the following:

Ex. 6: (Break Even problem) Find the number of units, $x$, that must be sold to break even given the Cost and Revenue functions for a business to be:

$$C(x) = 12x + 15,000$$
$$R(x) = 32x$$

Solve this system.

\[
\begin{align*}
y &= 12x + 15,000 \\
y &= 32x
\end{align*}
\]

$$32x = 12x + 15,000$$

$$20x = 15,000$$

$$x = 750$$ units.
Ex. 7: Baskets, Inc., is planning to introduce a new woven basket. The company estimates that $500 worth of new equipment will be needed to manufacture this new type of basket and that it will cost $15 per basket to manufacture. The company also estimates that the revenue from each basket will be $31.

a) Determine the Revenue function \( R(x) \) for \( x \) baskets:

\[
R(x) = 31x
\]

b) Determine the cost function \( C(x) \) for manufacturing \( x \) baskets.

\[
C(x) = 15x + 500
\]

c) Find the break-even point. Round up to the nearest whole basket.

\[
\begin{align*}
31x &= 15x + 500 \\
16x &= 500 \\
x &= \frac{500}{16} \\
x &= 31.25
\end{align*}
\]

He would need to sell 32 baskets.
Ex. 8: (A Geometry problem) The length of a garden is 8 feet longer than its width. The perimeter of the garden is 54 feet. Find the area of the garden.

\( y = (x + 8) \)

\[ 54 = 2y + 2x \]

\[ 54 = 2(x + 8) + 2y \]

\[ 54 = 2x + 16 + 2y \]

\[ 54 = 4x + 16 \]

\( 4x = 38 \)

\( x = 9.5 \text{ ft} \)

\( y = 17.5 \text{ ft} \)

Use this to find the area.

\[ A = 166.25 \text{ ft}^2 \]
Ex. 9: (Another Geometry problem) A rectangular lot whose perimeter is 320 feet is fenced along three sides. An expensive fencing along the lot's length cost $16 per foot, and an inexpensive fencing along the two side widths costs $5 per foot. The total cost of the fencing along the three sides comes to $2140. What are the lot's dimensions?

\[ 2l + 2w = 320 \]
\[ 16l + 5w + 5w = 2140 \]
\[ 16l + 10w = 2140 \]
Ex. 10: (A motion problem where current affects the rate) A boat travels 18 miles down a river in 1 hour. A return trip against the current takes $1\frac{1}{2}$ hours. Find the average speed of the boat in still water and the rate of the current.

\[
\begin{align*}
D &= r \cdot t \\
&= 18 \text{ mi} \quad \text{in} \ 1 \text{ hr} \\
X + y &= \text{rate of boat in still water, mph} \\
y &= \text{rate of current, mph}
\end{align*}
\]

\text{Downstream rate: } (X + y)

\text{Upstream rate: } (X - y)

\[
\begin{align*}
18 &= (X + y) \cdot 1 \\
18 &= (X - y) \cdot 1.5
\end{align*}
\]

\[
\begin{align*}
X - 18 &= y \\
1.5(18 - y) - 1.5y &= 18 \\
27 - 1.5y - 1.5y &= 18 \\
27 - 3y &= 18 \\
-3y &= -9 \\
y &= 3 \text{ mph}
\end{align*}
\]

\[
\begin{align*}
X + 3 &= 18 \\
X &= 15
\end{align*}
\]

The rate of the boat in still water is 15 mph, and the rate of the current is 3 mph.