Section 7.3 Logarithmic Functions and Their Graphs

Solve: a) $2^x = 8$  
\[ x = 3 \]  b) $2^x = 9$

Hmmmm..we need to be able to "undo" this exponential function. Let's find the inverse:

The Logarithmic Function is the inverse of the exponential function.

Definition: $b > 0$ is a fixed, real, positive number and $b \neq 1$. The logarithmic function with base = $b$ is

\[ y = \log_b x \quad \text{where} \quad b^y = x \]  is equivalent to the log. expression.

Note that these are inverses of each other! Also note that "$b" represents the base for the log or the base for the exponent.

Practice switching from log. form to exponential form and vice-versa:

\[ 2^x = 9 \quad \leftrightarrow \quad x = \log_2 9 \]
\[ \log_{13} x = 4 \quad \leftrightarrow \quad x = 13^4 \]
\[ x = 81 \]
Graphs of Log. Functions:

Recall graphs of inverses. It is usually easier to get points for the exponential graph and then interchange the coordinates to graph the log. graph.

Or you can use the basic information about their graphs to sketch and then use rules for transformations to shift, reflect, etc.

**Exponential Functions Graphs**
- Domain: All Real numbers
- Range: $(0, \infty)$
- Y-intercept: $(0,1)$
- Horizontal Asymptote $y=0$ (x-axis)
- Increasing if $b>1$
- Decreasing if $0<b<1$
- 1-1 function

**Logarithmic Functions Graphs**
- Domain: $(0, \infty)$
- Range: All Real numbers
- X-intercept: $(1,0)$
- Vertical Asymptote: $x=0$ (y-axis)
- (same)
- (same)
- (same)

Sketch $f(x) = 3^x$ and $g(x) = \log_3 x$
Practice:

a) Sketch $f(x) = \log_{\frac{1}{2}} x$

b) Sketch $g(x) = \log_2(x+4)$
Some Basic Properties of Logarithms:

1) \( \log_b 1 = 0 \) for any base \( b \)

2) \( \log_b b = 1 \) for any base \( b \)

3) \( \log_b b^x = x \) (inverse property)

4) \( b^{\log_b x} = x \) (inverse property)

Examples:

a) \( \log_{25} 25 = 2 \)  
   Evaluate  
   \[ \log_{25} 25 = 2 \]

b) \( \log_{\frac{1}{2}} 2 = -1 \)  
   \[ \log_{\frac{1}{2}} 2 = -1 \]

c) \( \log_{\sqrt{10}} \sqrt{10} = 1 \)  
   \[ \log_{\sqrt{10}} \sqrt{10} = 1 \]

d) \( \log_{10} 1 = 0 \)  

f) \( \log_{10} (100) \)  
   \[ \log_{10} 100 = 2 \]
With Variables:

a) \( \log_6 (2x) = -1 \)
   \[ 2x = 6^{-1} \]
   \[ x = \frac{1}{12} \]

b) \( \log_2 8^x = 5 \)
   \[ 8^x = 2^5 \]
   \[ 2^{3x} = 2^5 \]
   \[ 3x = 5 \]
   \[ x = \frac{5}{3} \]

c) \( \log_{3x} 2 \cdot \log_{3x} 2^2 = 2 \)
   \[ \log_{3x} 2 = \log_{3x} 2^2 \]
   \[ 3x = 3 \]
   \[ x = 1 \]
Common Logarithms have base $= 10$ such that
\[ \log_{10} x = \log x \quad \text{(see calculator)} \]

Natural Logarithms have natural base $= e$ such that
\[ \log_e x = \ln x \quad \text{(see calculator)} \]

Examples:
\[
\begin{align*}
\ln \sqrt[3]{e} &= \frac{1}{3} \\
\ln e^{\frac{1}{3}} &= \frac{1}{3} \\
\log_{10} 1000 &= 3 \\
\log_{10} 10^3 &= 3 \\
\log (10.5) &= \ln (4.78) \\
&\quad \text{(calculator)}
\end{align*}
\]