Section 1.5 - Relations and Functions

**Objectives:**

- To identify independent and dependent variables in a relation.
- To understand and use the definition of a function.
- To differentiate between function and non-function relationships when using a table of values, a graph and a word description.
- To identify and state the domain and range of a function.
- To identify a function as increasing, decreasing, or constant.
Vocabulary

• Relation:
  A rule that relates one set of numbers to another.

• Function:
  A relation where each input is assigned exactly one output.

• Independent Variable:
  Variable (usually "x") used to represent input values.

• Dependent Variable:
  Variable (usually "y") used to represent output values.

• Domain:
  The set of all input values. \% or interval

• Range:
  The set of all output values. \% or interval
Example: - Determine what values are in the domain and range.

- **Domain**
  - X-values (Left → Right)
  - Domain: \((-\infty, \infty)\)

- **Range**
  - y-values (Lowest → Highest)
  - Range: \([4, \infty)\)

- **Domain**
  - \((-\infty, \infty)\)

- **Range**
  - \((-\infty, \infty)\)

- **Domain**
  - \([-3, 3]\)

- **Range**
  - \([-3, 3]\)
Example 2 - Identify the independent and dependent variables. Determine the domain and the range. Does the table represent a function?

<table>
<thead>
<tr>
<th>x (input)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (output)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

- **indep. var. is x**
- **dep. var. is y**

> Domain: \(\{2, 4, 6, 8\}\)
> Range: \(\{1, 2, 3, 4\}\)

> *yes, function*

<table>
<thead>
<tr>
<th>x (input)</th>
<th>7</th>
<th>2</th>
<th>9</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (output)</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

- **indep. var. is x**
- **dep. var. is y**

> Domain: \(\{0, 2, 7, 9\}\)
> Range: \(\{-3, 1, 4\}\)

> Input (x) has no repeat values,

> *yes, a function*

<table>
<thead>
<tr>
<th>x (input)</th>
<th>0</th>
<th>-1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (output)</td>
<td>4</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

- **indep. var. is x**
- **dep. var. is y**

> Domain: \(\{-1, 0, 1\}\)
> Range: \(\{3, 4, 5, 6\}\)

> Not a Function
Example: On Monday, Laura ran 3 miles in 25 minutes. On Tuesday, she ran 2 miles in 17 minutes. On Thursday, she ran 3 miles in 28 minutes. Is time a function of the distance run? (Hint: use a table to organize your information).

<table>
<thead>
<tr>
<th>ind. var</th>
<th>dep var</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance</td>
<td>time</td>
</tr>
<tr>
<td>mts</td>
<td>min</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
</tr>
</tbody>
</table>

Not a function
X = 3 has 2 different output values.

Is the distance run a function of the time?

<table>
<thead>
<tr>
<th>ind. var</th>
<th>dep var</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>distance</td>
</tr>
<tr>
<td>min</td>
<td>mts</td>
</tr>
<tr>
<td>25</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>3</td>
</tr>
</tbody>
</table>

Yes, this is a function because no input values are repeated.
Vertical Line Test - a relation is not a function if a vertical line could be drawn that intersects the graph in more than one point. Otherwise the relation is a function.

(Use previous examples and those below.)
Example: A Verbal Description:

Jamie buys apples at a price of $1.29 per pound. The total cost he pays is related to the number of pounds of apples he buys.

Is this a relation?  
Yes

What is the independent variable?  
\( n = \# \text{ pounds of apples} \)

What is the dependent variable?  
\( C = \text{total cost of apples} \)

Is this a function?  
Yes  
\[ C = 1.29n \]

Hint: These phrases indicate relations, and the DEPENDENT variable is always the first one mentioned: 
"is dependent on"  
"is a function of"  
"is related to"
Increasing, Decreasing, and Constant Functions -

As the value of the independent variable increases, if the value of the dependent variable:

- ALWAYS increases, the function is **Increasing**.
- ALWAYS decreases, the function is **Decreasing**.
- remains the same, the function is **Constant**.

Some functions do not fit into only one of the above so we describe them as a **combination**.

Graphically this means:
Example: Identify whether each function is increasing, decreasing, constant or a combination of these.