Section 3.5 - Exponential Functions

Definition of an Exponential Function

- An exponential function is a function that can be represented by the equation

$$f(x) = ab^x$$

where a and b are constants, $b > 0$ and $b \neq 1$.

- The independent variable is in the exponent.

- **Ex.** $f(x) = 2^x$ **is** an exponential function,

but $f(x) = x^2$ **is not**, because the variable is not in the exponent.

Ex.1 - Identify the equations of exponential functions. For those that are exponential, identify *a* and *b*. $f(x) = ab^x$

a.) $f(x) = (\frac{1}{3})^x = (1)(\frac{1}{3})^x$

yes, exponential a b

$a = 1$ $b = \frac{1}{3}$

b.) $f(x) = \frac{1}{3}x$

not exponential

c.) $f(x) = 5(3)^x$

yes. exp function

$a = 5$ $b = 3$

d.) $f(x) = 4(\frac{1}{2})^x$

yes, exp. function

$a = 4$

$b = \frac{1}{2}$

e.) $f(x) = 7x^3$

not exp. function

Identifying Exponential Functions from a Table

- A function is said to be an exponential function if equal steps in the independent variable produce equal ratios for the dependent variable.

- Ex. Does the following table represent an exponential function?
If so, is the function a decay or growth model?

decreasing   increasing

+1   +1   +1

yes, exponential growth

x	-2	-1	0	1	2	3
f(x)	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

3   3   3   3   3

$\frac{1}{3} \div \frac{1}{9}$ $1 \div \frac{1}{3}$

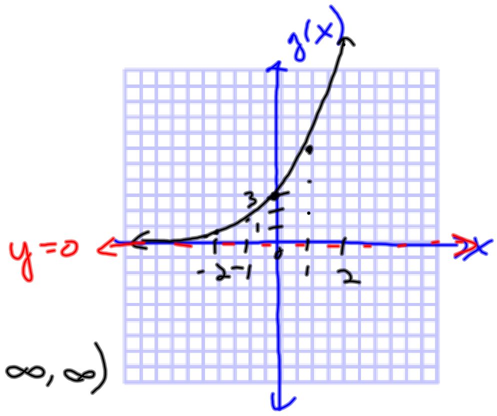
$\frac{1}{3} \cdot \frac{9}{1} = \frac{9}{3} = \boxed{3}$ $1 \cdot \frac{3}{1} = \boxed{3}$

Graphing Exponential Functions

Ex.2 - Make a table of values for each exponential function. Plot the points and sketch the graph. State the domain and the range.

a.) $g(x) = 3 \cdot (2)^x$

	x	$g(x)$
	-2	$3 \cdot 2^{-2} = 3 \cdot \frac{1}{2^2} = \frac{3}{1} \cdot \frac{1}{4} = \boxed{\frac{3}{4}}$
	-1	$3 \cdot 2^{-1} = 3 \cdot \frac{1}{2^1} = \boxed{\frac{3}{2}}$
*	0	$3 \cdot 2^0 = 3 \cdot 1 = \boxed{3}$
	1	$3 \cdot 2^1 = 6$
	2	$3 \cdot 2^2 = 3 \cdot 4 = 12$



Domain $(-\infty, \infty)$

Range $(0, \infty)$

Ex.2 continued - Make a table of values for each exponential function. Plot the points and sketch the graph. State the domain and the range.

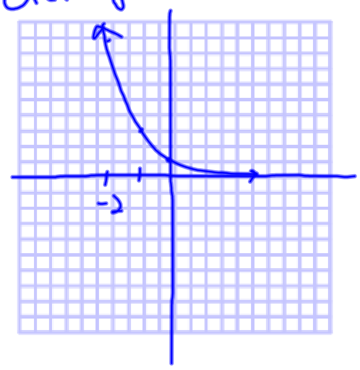
b.) $g(x) = (\frac{1}{3})^x$

x	$g(x)$
-2	$(\frac{1}{3})^{-2} = 3^2 = 9$
-1	$(\frac{1}{3})^{-1} = 3^1 = 3$
* 0	$(\frac{1}{3})^0 = 1$
1	$\frac{1}{3}^1 = \frac{1}{3}$
2	$(\frac{1}{3})^2 = \frac{1^2}{3^2} = \frac{1}{9}$

domain
$(-\infty, \infty)$

range $(0, \infty)$

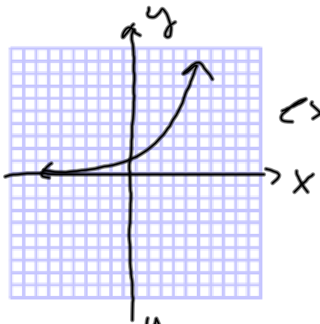
decay



c.) $h(x) = 2(\frac{1}{3})^x$ (try on calculator...look at table)

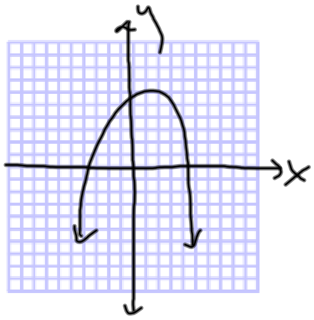
Ex.3 - Identify which graphs represent exponential functions. For those that are exponential, tell whether they represent growth or decay.

a.)



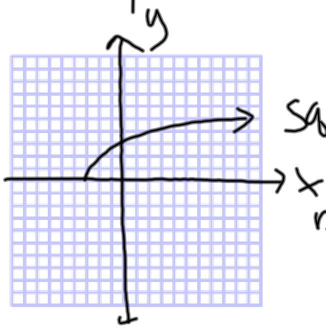
exp growth

b.)



not exponential

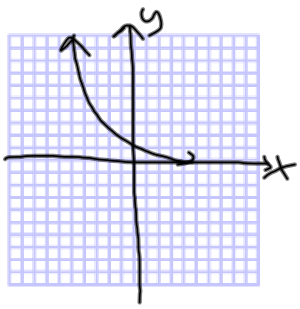
c.)



Sq. root funct.

not exp

d.)



exp. decay

Characteristics of Exponential Functions:

- For the exponential function $f(x) = ab^x$ and $a > 0$:

  - Same shape - one end steep and the other seems almost flat
  - Domain : $(-\infty, \infty)$ all Real Numbers
  - Range : $(0, \infty)$ (range relates to horizontal asymptote)
  - Horizontal Asymptote: - horizontal line that the graph gets really really close to, but never reaches and never intersects
    x-axis $\boxed{y=0}$
  - Vertical Intercept (0, a)
  - Increasing (growth) if b >1
  - Decreasing (decay) if 0 < b < 1

Ex.4 - Determine whether each table represents an exponential function. If it is exponential, give the common ratio and decide if it represents exponential growth or decay.

a.)

x	0	1	2	3	4	5
f(x)	1	3	9	27	81	243

y_1 y_2 y_2 y_3 y_3

3 3 3 3 3 $= b$

$\frac{y_2}{y_1} = \frac{3}{1} = \boxed{3}$ $\frac{y_3}{y_2} = \frac{9}{3} = \boxed{3}$

yes, exponential growth

b.)

x	-4	-2	0	2	4	6
f(x)	$\frac{3}{16}$	$\frac{3}{4}$	3	12	48	192

4 4 4 4 4 $\boxed{b=4}$

$\frac{3}{4} \div \frac{3}{16}$

$= \frac{3}{4} \cdot \frac{16}{3} = \frac{16}{4} = 4$

$3 \div \frac{3}{4}$

$= 3 \cdot \frac{4}{3} = \frac{12}{3} = 4$

exp. growth

c.)

x	-3	-2	-1	0	1	2
f(x)	16	8	4	2	1	$\frac{1}{2}$

y_1 y_2 y_3 y_4

$8 \div 16 = \frac{1}{2}$ $\boxed{\frac{1}{2}}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$\frac{4}{8} = \frac{1}{2}$

$\boxed{b=\frac{1}{2}}$ exp. decay

d.)

x	-2	0	2	4	6	8
f(x)	-1	3	7	11	15	19

+4

$\frac{3}{-1} = -3$

not exp → linear !

$\frac{7}{3} = 2\frac{1}{3}$

Writing Exponential Functions

- Lets look back at the graphs and compare them with their equations. What do you notice about the vertical intercept?

$(0, a)$

- Lets look back at the tables and compare them with their equations. What do you notice about the common ratio?

$= b$

Exponential Functions:

- A function that can be represented by the equation

$f(x) = ab^x$ for $b > 0$ and $b \neq 1$,

where *a* is the vertical intercept,
and the base, *b*, is the common ratio *(assumed to be based on steps of 1).*

Ex.5 - Find the equation for the exponential function represented by the table. Identify whether the equation represents a growth or decay function.

a.)

x	-2	-1	0	1	2
f(x)	$\frac{2}{9}$	$\frac{2}{3}$	2	6	18

3 (0,2) 3 3

vert. int

$\boxed{a=2}$

find b

$\frac{6}{2} = \boxed{3 = b}$

$f(x) = ab^x$

$f(x) = 2(3)^x$

growth

b.)

x	-2	-1	0	1	2
f(x)	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$

$\frac{4}{16} = \boxed{\frac{1}{4}}$ $\boxed{\frac{1}{4}}$ $\frac{1}{4} \div 1 = \boxed{1/4}$

$a = 1$

$b = 1/4$

$f(x) = 1(1/4)^x = .25^x$

decay

c.)

x	-2	-1	0	1	2
f(x)	25	10	4	$\frac{8}{5}$	$\frac{16}{25}$

vert int

$\frac{10}{25} = \frac{2}{5}$ $\frac{4}{10} = \frac{2}{5} = .4$

$a = 4$

$b = 2/5$ or $.4$

$f(x) = 4(2/5)^x$

decay