

## 4.2 Poisson Random Variable - How is it different from the Binomial?

**Binomial Random Variable**

Discrete Random Variable  
 Independent Trials  
 Fixed Number of Trials  
 Probability of a Success Does NOT Change

We are interested in  $X = \#$  of Successes in the  $n$  trials

**Poisson Random Variable**

Discrete Random Variable  
 The average *rate* of occurrence of a success is constant.  
 Events occur independently of the last occurrence.

We are interested in  $X =$  Number of occurrences in an interval of time or space

**Poisson:** An average rate is given...  
 a number is given per unit (time, area, volume).

You have interest in  $x$  successes in a specific sample.

Then...  $\lambda =$  **expected** number per sample

So to find probabilities...

**NOTE:** For a Binomial situation with large  $n$  and small  $p$ , the Poisson distribution gives a good approximate probability.  
 Just let  $\lambda = np$ .

What clues will there be that you should use Binomial or Poisson?

**Binomial:** An exact probability of a success is given or implied.  
You have interest in  $x$  successes out of  $n$  trials.

**Poisson:** An average rate is given.  
You have interest in  $x$  successes in a specific sample.

### **Binomial or Poisson?**

Light bulbs are packaged in boxes of sixty. The manufacturer claims the probability a single light bulb is defective is 0.09. What is the probability a box of sixty bulbs contains five defectives?

## Binomial or Poisson?

You notice that your computer crashes on average once every five days. What is the probability it will crash three times in one week?

**Example:**

A tent manufacturer has an established flaw rate of 1 flaw per 50 square feet of canvas.

A single tent made of canvas uses 280 square feet.



Find the probability there are no flaws in a tent.

Find the probability there are more than four flaws in a tent.

**Example:**  
**Yum Cookie Company produces huge volumes of cookie dough at one time. A vat of dough will make 10,000 cookies. 40,000 chocolate chips are added to the dough and thoroughly mixed.**



Find the probability there are no chocolate chips in one cookie.

Find the probability there are at least 7 chocolate chips on one cookie.

Poisson  $\lambda = 4$

x	PDF $P(X=x)$	CDF $P(X \leq x)$
0	0.018316	0.01832
1	0.073263	0.09158
2	0.146525	0.23810
3	0.195367	0.43347
4	0.195367	0.62884
5	0.156293	0.78513
6	0.104196	0.88933
7	0.059540	0.94887
8	0.029770	0.97864
9	0.013231	0.99187
10	0.005292	0.99716
11	0.001925	0.99908
12	0.000642	0.99973
13	0.000197	0.99992
14	0.000056	0.99998
15	0.000015	1.00000
16	0.000004	1.00000
17	0.000001	1.00000

**Example:**  
**Roughly 0.5% of the human population has lupus. A random sample of 4000 people is taken. Find the probability 25 people have lupus.**

**Example:**

It is accepted that there is an average of 20 maggots per 100 grams of canned mushrooms. You purchase a 113 gram can of mushrooms. Find the probability you get fewer than 5 maggots.



**Example:**

Traffic engineers observe that an average of 7.4 cars fail to stop for a red light at a particular intersection (between 7am and 8pm each weekday). The public wants cameras installed to ticket offenders. Find the probability at least 40 cars fail to stop between 7am and 8pm in a typical five day work week.

