Name: _____KEY_____

Probability/Basic Set Theory Worksheet

1. Find the union $A \cup B$ and the intersection of $A \cap B$ of the two sets A and B, where:

a. $A = \{0, 1, 2\}\ B = \{2, 3, 4\}$

$$A \cup B = \{0,1,2,3,4\}, \qquad A \cap B = \{2\}$$

b. $A = \{x|\ 0 < x < 2\}, B = \{x|\ 1 \leq x < 3\}$

$$A \cup B = \{x|\ 0 < x < 3\}, \qquad A \cap B = \{x|\ 1 \leq x < 2\}$$

2. Find the compliment $A^c$ of the set $A$ with respect to the space $\boldsymbol{S}$ if:

a. $\boldsymbol{S} = \{x|\ 0 < x < 1\},\ A = \{x|\frac{5}{8} < x < 1\}$

$$A^c = \left\{x|\ 0 < x \leq \frac{5}{8}\right\}$$

b. $\boldsymbol{S} = \{(x, y, z)|\ x^2 + y^2 + z^2 \leq 1\},\ A = \{(x, y, z)|\ x^2 + y^2 + z^2 = 1\}$

$$A^c = \{(x, y, z)|\ x^2 + y^2 + z^2 < 1\ \}$$

3. A random experiment consists of drawing a card from an ordinary deck of 52 playing cards. Let $A$ denote the collection of the 13 hearts and let $B$ denote the collection of the 4 kings. Compute $P(A), P(B), P(A \cap B), and\ P(A \cup B)$.

$$P(A) = \frac{1}{4}$$

$$P(B) = \frac{1}{13}$$

$$P(A \cap B) = \frac{1}{52}$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{4}{13}$$

4. If the sample space is $\boldsymbol{S} = A \cup B$ and if $P(A) = 0.8$ and $P(B) = 0.5$, find $P(A \cap B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$P(A \cap B) = 0.8 + 0.5 - 1 = 0.3$$

5. Out of the students in class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

$$P(A \cap B^c) = .6 - .4 = 0.2$$

$$P(B \cap A^c) = .7 - .4 = 0.3$$

$$P(A \cup B) = 0.2 + 0.3 + 0.4 = 0.9$$

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - 0.9 = 0.1$$

6. Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = 0.5$, $P(B) = 0.4$, and $P(A \cap B) = 0.25$.

a. Compute the probability that the selected individual has at least one of the two types of cards.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.4 - 0.25$$

$$= 0.65$$

b. What is the probability that the selected individual has neither type of card?

$$1 - P(A \cup B) = 1 - 0.65 = 0.35$$

c. Calculate the probability that the selected student has a Visa card but not a MasterCard.

$$P(A \cap B^c) = 0.5 - 0.25 = 0.25$$

7. A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Find the probability that the outcome of one roll is less than 4.

$$P(outcome < 4) = \frac{4}{9} = 0.\overline{4}$$

8. A utility company offers a lifeline rate to any house-hold whose electricity usage falls below 240 kWh during a particular month. Let A denote the event that a randomly selected household in a certain community does not exceed the lifeline usage during January, and let B be the analogous event for the month of July. Events A and B refer to the same household. Suppose that $P(A) = 0.8$, $P(B) = 0.7$, and $P(A \cup B) = 0.9$. Compute the following:

a. $P(A \cap B)$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.8 + 0.7 - 0.9 = 0.6$$

b. The probability that the lifeline usage amount is exceeded in exactly one of the two months. Describe this event in terms of A and B.

$$P(A \text{ but not } B) + P(B \text{ but not } A) = P(A \cap B^c) + P(B \cap A^c)$$

$$= 0.8 - 0.6 + 0.7 - 0.6$$

$$= 0.3$$