Chapter 3
What are the chances? A Lesson in Probability

Where are we headed...

Make an assumption about a population. I think the population looks like _____ or I think the population behaves a specific way.

Collect data in a representative way.

Study the sample. If our population is the way we think it is then we expect our sample to look a certain way. If the sample is unexpected then we can calculate how likely it is that would observe such a sample.

If the probability is very small, we conclude the assumption about the way the population looks is wrong.

Hypothesis Testing

Probability is the measure of how likely an event is.

Vocabulary:

Experiment: A process that results in outcomes that can't be predicted in advance with certainty.

Sample Space: A set of all possible outcomes for an experiment.

Event: A subset of the sample space.

The Probability of an event: $0 \leq P(A) \leq 1$

The Probability of the sample space:

$$P(S) = 1$$

Flip a coin:

Exp: Flip a coin once.

Sample Space: {H, T}

Event: A = Observing Heads

$P(A) = 1/2, .5, 50\%$

Roll one die:

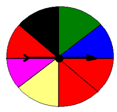
Experiment: Roll a die once.

Sample Space: {1, 2, 3, 4, 5, 6}

Event: A = We observe an event >4

$P(A) = 2/6$ OR $1/3$

Spin the spinner:



Exp: Spinning the spinner once.

SS: {Red, Blue, Yellow, Green, Black, Pink}

Event: A = Getting Red

$P(A) = 3/8$

Example:

Experiment - Have two children

<u>Sample Space:</u>

{GG, BB, GB, BG}

The likelihood of each outcome is $1/4$.

Events:

A = having two girls

B = having at least one boy

C = having exactly one boy

Probabilities of Events:

P(A) = $1/4$

P(B) = $3/4$

P(C) = $1/2$

We can combine events...

Union $P(A \cup B)$

A or B or both.

Intersection $P(A \cap B)$

A and B.

Complement A^c

NOT A!

Example:

Experiment: Pick ONE card.

Sample Space: {A clubs, 2 clubs, ...}

P(red) $= 26/52 = 1/2$

P(face) $= 12/52 = 3/13$

P(red and face) $= 6/52$

P(black and number) $= 20/52$

P(red and black) $= 0$

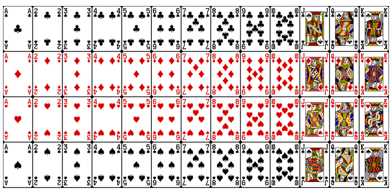
P(not an Ace) $= 48/52$

$= 1 - P(\text{Ace})$

$= 1 - 4/52$

$= 48/52$

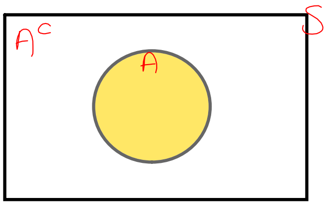
	face	number	
red	6	20	26
black	6	20	26
	12	40	52



Venn Diagrams help visualize probability situations.

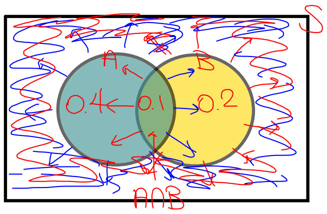
We can create new events by combine other events.

Let A and B represent events.



$P(A) = 0.4$

$P(A^c) = 0.6 = 1 - P(A)$



$P(A) = 0.5$

$P(B) = 0.3$

$P(A \cap B) = 0.1$

$P(A \cap B^c) = 0.4$

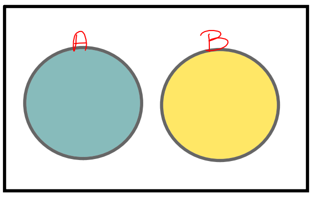
$P(B \cap A^c) = 0.2$

$P(A \cup B) = 0.7$

$= P(A) + P(B) - P(A \cap B)$

$P(A^c \cap B^c) = 0.3$

$= 1 - P(A \cup B)$



$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$

Summary:

Complementary Events

P(A) + P(A^c) = 1

P(A^c) = $1 - P(A)$

P(A) = $1 - P(A^c)$

If A and B are mutually exclusive, then

P(A$\cap$B) = 0

P(A$\cup$B) = $P(A) + P(B) - \overbrace{P(A \cap B)}^{0}$

If A and B can occur at the same time then P(A$\cap$B)$\neq$0, so

P(A$\cup$B) = $P(A) + P(B) - P(A \cap B)$

Example: Suppose P(A) = 0.5 and P(B)=0.4.

(1) If events A and B are mutually exclusive, then find P(A ∪ B).

$P(A \cup B) = 0.9$

(2) If the probability events A and B happen at the same time is 0.1, then find P(A ∪ B).

$P(A \cup B) = 0.8$

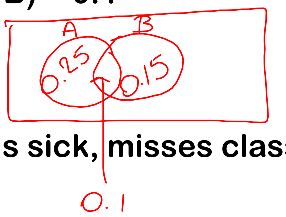
(3) Find the probability event B does not occur.

$P(B^c) = 1 - P(B) = 0.6$

Example: A is the event a student is sick. B is the event a student misses class.

P(A) = 0.35, P(B) = 0.25, P(A∩B) = 0.1

Draw Venn Diagrams



Find the probability a student is sick, misses class, or both.

$P(A \cup B) = 0.5$

Find the probability the student is neither sick nor misses class.

$1 - P(A \cup B) = 0.5$

Find the probability the student is sick but does not miss class.

$P(A \cap B^c) = 0.25$

Example: Roll two dice. + Sum

First Roll / Second Roll

Second Roll \ First Roll	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Sample Space? $\{2, 3, 4, \ldots, 12\}$

P(Prime or Even) $= 1 - P(\text{Not Prime and Even})$

Not

$= 1 - 4/36$

P(Greater than 3) $= 32/36$

$= 1 - P(\text{Less than or equal to } 3) = 1 - 3/36$

$= 33/36$

Example:
A die is constructed so that a 6 occurs twice as often as a 5, which occurs three times as often as a 1, 2, 3, or 4. Roll the die once.

List the Sample Space: $\{1, 2, 3, 4, 5, 6\}$

Find the probability of observing each outcome in the sample space:

$x + x + x + x + 3x + 6x = 1$

$13x = 1 \quad x = 1/13$

$P(1)$ thru $P(4) = 1/13$

$P(5) = 3/13$

$P(6) = 6/13$

P(observe a 5)

$P(5) = 3/13$

P(observe a number less than 5)

$= 4/13$

We will select a person at random.
Calculate the following probabilities.

	Physics	Math
Male	43	52
Female	39	49

P(male) = $\frac{43+52}{43+52+39+49}$

P(math) = $\frac{52+49}{43+52+39+49}$

P(male and math) = $\frac{52}{183}$

P(male or math) = $\frac{43+52+49}{183}$

P(male or female) = 1