Continuous Random Variables can take on all values over an interval.

Outcomes are measured

Since we are working with Continuous Random Variables, we could be dealing with measurements such as:

Lengths
Weights
Masses

We have a function (or a way of finding the function) that describes the probability structure...

$$\int_a^b f(x)\,dx = P(a \le X \le b)$$

$$\mu_X = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\sigma_X^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu_X^2$$

$$\sigma_X = \sqrt{\sigma_X^2}$$

Let X be the # of hours a reserve book is checked out. The limit that the book can be checked out is 2 hours.

$$f(x) = \begin{cases} 0.375x^2 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$

Find the probability a book is checked out from 1 to 2 hours.

$$P(1 \le X \le 2) = \int_1^2 0.375x^2\,dx$$

$$= \left[0.375 \cdot \frac{x^3}{3}\right]_1^2$$

$$= 0.875$$

Find μ_X and σ_X.

$$\mu_X = \int_0^2 x \cdot 0.375x^2\,dx$$

$$= \left[0.375 \frac{x^4}{4}\right]_0^2$$

$$= 0.375 \cdot 4$$

$$= 1.5 \text{ hours}$$

$$\sigma_X^2 = \int_0^2 x^2\, 0.375x^2\,dx - 1.5^2$$

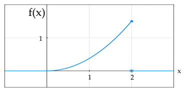
$$\sigma_X^2 = 0.15 \text{ hrs}^2$$

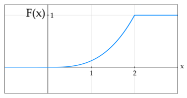
$$\sigma_X = 0.387 \text{ hrs.}$$

For a continuous random variable, X, we can create a cumulative function that we call F(x) that will sum up probability to specific x values.

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$$

$$f(x) = \begin{cases} 0.375x^2 & 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$





What is $F(x)$?

$$F(x) = \int_0^x 0.375t^2\,dt$$

$$= \left[0.375\frac{t^3}{3}\right]_0^x$$

$$= 0.375\frac{x^3}{3} - 0$$

$$F(x) = \begin{cases} 0 & x<0 \\ x^3/8 & 0 \le x \le 2 \\ 1 & x>2 \end{cases}$$

How could we use this information to find the median?

$$F(x) = 0.5$$

$$\frac{x^3}{8} = \frac{1}{2}$$

$$x^3 = 4$$

$$\sqrt[3]{x^3} = \sqrt[3]{4}$$

$$x \approx 1.59 \text{ hours}$$

Suppose a random variable, X, represents the time in milliseconds for a reaction to be completed.

The probability function (or distribution) is given by:

$$f(x) = 0.05e^{-0.05x} \quad if\ x \ge 0$$

Find the probability that the reaction is completed in 40 milliseconds.

$$P(X<40) = \int_0^{40} 0.05e^{-0.05x}\,dx$$

$$= \left[-e^{-0.05x}\right]_0^{40}$$

$$= -e^{-2} + 1$$

$$= 1 - \frac{1}{e^2}$$

$$\approx 0.865$$