

Section 3.3 Random Variables

Think about the experiments we have worked with so far...

Roll a die.

Select a person and measure height.

Flip a coin.

Select a component from a production run to see if it works.

A **Random Variable** is a function.

Outcome \xrightarrow{X} Numerical Value

Random Variables can be **DISCRETE** or **CONTINUOUS**.

Discrete Random Variables can take on only distinct values.

Outcomes are counted ex. # of objects

Continuous Random Variables can take on all values over an interval.

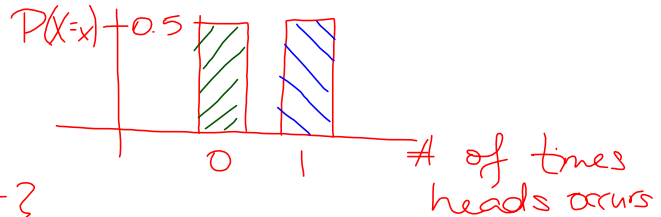
Outcomes are measured ex. height
weight

Example: Flip a coin once

$$SS: \{H, T\}$$

$X = \#$ of times heads occurs.

x	$P(X=x)$
0	0.5
1	0.5

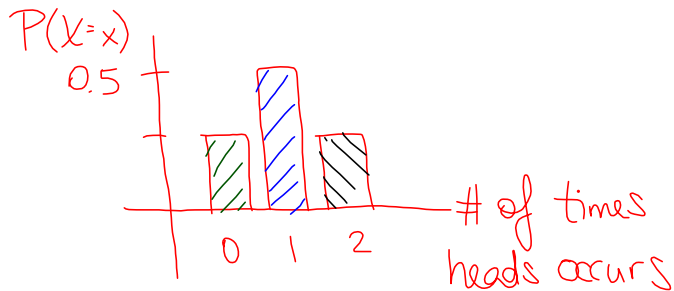


Example: Flip a coin twice.

$$SS: \{HH, HT, TH, TT\}$$

$X = \#$ of times heads occurs

x	$P(X=x)$
0	0.25
1	0.5
2	0.25

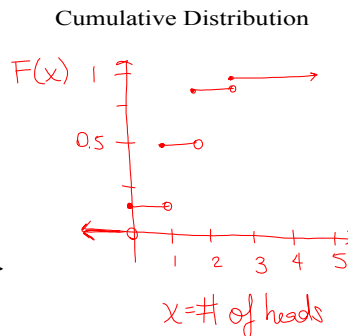
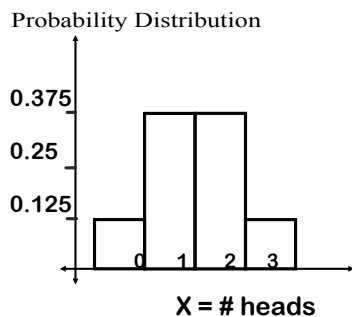


Example: Flip a coin three times.

$$\text{Sample Space: } \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$X = \#$ of times heads occurs

x	$P(X=x)$
0	$1/8$ or 0.125
1	$3/8$ or 0.375
2	$3/8$ or 0.375
3	$1/8$ or 0.125



Recall the **Sample Mean**

$$\bar{x} = \frac{\sum x_i}{n}$$

When dealing with Random Variables:

The mean of a random variable is the mean of a hypothetical sample that follows the probability distribution perfectly.

Mean of a Random Variable

Measure of the center of the distribution

Expected Value of X = E(X)

$$E(X) = \mu_x = \sum x_i P(X=x_i)$$

$$\text{OR } \sum x P(x)$$

Example: Let X represent the number of emergency calls at LBCC in a week. Find the mean.

X	$P(X=x)$
0	0.40
1	0.30
2	0.15
3	0.08
4	0.07

$$\begin{aligned} \mu_x &= 0 \cdot 0.4 + 1 \cdot 0.3 + 2 \cdot 0.15 \\ &\quad + 3 \cdot 0.08 + 4 \cdot 0.07 \\ &= 1.12 \end{aligned}$$

Variance of a Random Variable:**Measure of the spread of the distribution****Variance of a sample that follows the distribution perfectly**

$$\begin{aligned}\sigma_x^2 &= \sum (x_i - \mu_x)^2 \cdot P(X = x_i) \\ &= \left[\sum x_i^2 P(X = x_i) \right] - \mu_x^2\end{aligned}$$

Example: Let X represent the number of emergency calls at LBCC in a week. Find the variance.

x	$P(X = x)$
0	0.40
1	0.30
2	0.15
3	0.08
4	0.07

$$\sigma_x^2 = \left[\sum x_i^2 P(X = x_i) \right] - \mu_x^2$$

$$\mu_x = 1.12$$

$$\sigma_x^2 = 1.486$$

$$\sigma_x = 1.219$$