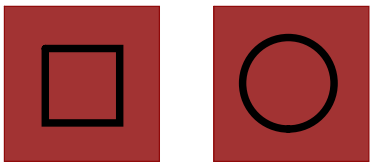
What is ESP?

Do you have extrasensory perception, ESP?



Can you determine which card is in your hand without looking at it?

What are we measuring?

$$\frac{\#\text{ of successes}}{\#\text{ of trials}}$$

What would you expect?

Our guess at the true proportion is 50%

6.3 Hypothesis Testing for Proportions

Binomial Situations

Random Variable: $\chi = \#$ of successes

Parameter of interest:

the true proportion: p

Distribution of Point Estimate:

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right) \quad \text{if } n \text{ is large enough}$$

Hypothesis Test

$H_0: p = p_0$

$H_1: p \neq p_0$

Test Requirements:

$n \cdot p_0 > 10$

$n \cdot q_0 > 10$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Do you have ESP?

$H_0: p \leq .5$
$H_1: p > .5$

$n \cdot p_0 = 26 \cdot 0.5 = 13 > 10$
$n \cdot q_0 = 13 > 10$

$$z = \frac{\frac{7}{26} - 0.5}{\sqrt{\frac{0.5(0.5)}{26}}} = -2.353$$

p-value: normcdf(-2.353, L.N., 0, 1)
$= 0.991$

Not enough evidence to reject H_0.

There is no evidence at all that the class has ESP, at least in this way. (p-value: 0.991)

A manufacturer claims that 90% or more readings taken by a machine are accurate. Is there evidence that the manufacturer is wrong?

$H_0: p \geq .9$
$H_1: p < .9$

$n = 150$
142 successes.

$z = 1.905$

p-value: 0.9716

Fail to reject H_0.

There is not enough evidence to suggest the manufacturer's claim is incorrect (p-value: 0.9716).

Section 6.4 Hypothesis Testing with Small Samples

Certain requirements must be fulfilled in order to use the hypothesis testing procedures:

Hypothesis Test for a Population Mean

Large enough $n \rightarrow$ Use z-score
Use s if σ is unknown.

Small $n \rightarrow$ If the pop. is normalish, use the t-dist.

Don't forget to see if your sample has outliers.

Example: Measurements of groundwater concentrations of silica, in mg/L, were made at a sample of 12 wells in a certain city. The sample mean concentration was 61.3 and the standard deviation was 5.2. Can you conclude that the mean concentration of silica is greater than 60 mg/L?

$n = 12 \quad \bar{x} = 61.3 \quad s = 5.2$

$H_0: \mu \leq 60$

$H_A: \mu > 60$

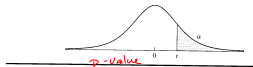
$$t = \frac{61.3 - 60}{5.2/\sqrt{12}} = 0.8666$$

p-value: tcdf(0.8666, L.N., 11) = 0.202

Fail to reject H_0.

There is not enough evidence to suggest the mean level of silica in the water is more than 60 mg/L. (p-value: 0.202)

TABLE A.3 Upper percentage points for the Student's t distribution



ν	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
				α					
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Let's perform the same hypothesis test using a calculator:

$\bar{x} = 61.3$ mg/L

$s_x = 5.2$ mg/L

$n = 12$

H_o: $\mu \leq 60$

H_1: $\mu > 60$

On the TI-84, go to...

Stat

Tests

T-Test

Inpt: Highlight "Stats"

Enter: μ_o
sample mean
standard deviation
sample size
specify alt hypothesis

Select Calculate

On the TI-89, go to...

You will need to access F6-Tests

T-Test

Data Input Method: Stats

Enter μ_o
sample mean
standard deviation
sample size
specify alt hypothesis

For Results, specify calculate

TI-84 Windows