Solution to Problems of Chapter 20 Online Homework Assignment

Q20.8. **Reason:** The following figure shows a representation of the charges on the metal sphere before and after the positively charged rod is brought near the neutral metal sphere.

(a) Notice that the positive charges on the metal sphere have not moved, just the electrons. Since the ball is metal, it is easy for the electrons to move. Electrons associated with nuclei on the region of the ball nearest the positive rod will move as close to the positive rod as possible but may not actually disassociate from the nucleus. In other regions of the metal ball, the electrons may actually leave their “mother” nucleus in order to get closer to the very positive rod. In regions of the ball far from the rod, some charge polarization may occur with the electrons “leaning” toward the positive rod but not disassociated from the nucleus. Finally, in regions farthest from the rod, the influence of the “mother” nucleus may be so much greater than that of the distant rod that the electrons essentially don’t feel the presence of the positively charged rod.

(b) Since the negative charges are concentrated on the region of the sphere closest to the rod, there will be a net attractive force of the sphere to the rod.

**Assess:** This question uses our knowledge of charge polarization, the manner in which like and unlike charges interact, and the fact that the electrons are the mobile charge carriers.

Q20.26. **Reason:** In the following figure, let’s label the charges 1, 2, and 3 so we can keep track of the electric field due to each of them. Let’s also represent the separation by $r$ as shown in the figure. Next, in order to make a quantitative comparison, define a unit of electric field strength as $E = kq/r^2$. For example, at a distance of $2r$ from a charge $q$, the magnitude of the field is $E = kq/(2r)^2 = (1/4)(kq/r^2) = (1/4)$ unit.
Note that in case A the electric field due to 2 and 3 add to zero and in case C the electric field due to 1 and 2 add to zero.

Assess: In this case the calculation is simplified by defining a unit of electric field and then determining the electric field at the dot due to each charge in terms of this definition. This is a useful technique when you want to compare values.

P20.6. Prepare: We will use the charge model and the model of a conductor as a material through which electrons move. An electron has a negative charge of magnitude $1.6 \times 10^{-19}$ C.

Solve: Because the metal spheres are identical, the total charge is split equally between the two spheres. That is, $q_A = q_B = 5 \times 10^{11}$ electrons. Thus, the charge on metal sphere A and B is $(5 \times 10^{11})(-1.6 \times 10^{-19} \text{ C}) = -80 \text{ nC}$.

Assess: Flow of charge from one charged conductor to another occurs when they come into contact.

P20.14. Prepare: Please refer to Figure P20.14. Charges A, B, and C are point charges. Charge A experiences an electric force $F_{B \text{ on } A}$ due to charge B and an electric force $F_{C \text{ on } A}$ due to charge C. The force $F_{B \text{ on } A}$ is directed to the right, and the force $F_{C \text{ on } A}$ is directed to the left.

Solve: Coulomb’s law yields:

$$F_{B \text{ on } A} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-6} \text{ C})(1.0 \times 10^{-8} \text{ C})}{(1.0 \times 10^{-2} \text{ m})^2} = 9.0 \times 10^{-5} \text{ N}$$

$$F_{C \text{ on } A} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-6} \text{ C})(4.0 \times 10^{-8} \text{ C})}{(2.0 \times 10^{-2} \text{ m})^2} = 9.0 \times 10^{-5} \text{ N}$$

The net force on A is

$$\vec{F}_{\text{on } A} = \vec{F}_{B \text{ on } A} + \vec{F}_{C \text{ on } A} = (9.0 \times 10^{-5} \text{ N}, +x\text{-direction}) + (9.0 \times 10^{-5} \text{ N}, -x\text{-direction}) = 0 \text{ N}$$

Assess: The force on A by C is the same (but in the opposite direction) as that of B on C because C has four times the charge and is twice the distance away compared to B. Check this statement against Coulomb’s law!

P20.24. Prepare: The electric field is that of the two charges placed on the y-axis. Please refer to Figure P20.24. We denote the upper charge by $q_1$ and the lower charge by $q_2$. Because both the charges are positive, their electric fields at P are directed away from the charges.

Solve: The electric field strength of $q_1$ is

$$E_1 = K \frac{|q_1|}{r_1^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \times 10^{-9} \text{ C})}{(0.050 \text{ m})^2} = 1800 \text{ N/C}$$

Similarly, the electric field strength of $q_2$ is

$$E_2 = K \frac{|q_2|}{r_2^2} = \frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \times 10^{-9} \text{ C})}{(0.050 \text{ m})^2} = 1800 \text{ N/C}$$

We will now calculate the components of these electric fields. The electric field due to $q_1$ is away from $q_1$ in the fourth quadrant and that due to $q_2$ is away from $q_2$ in the first quadrant. Their components are

$$E_{1x} = E_1 \cos 45^\circ$$
$$E_{1y} = -E_1 \sin 45^\circ$$
$$E_{2x} = E_2 \cos 45^\circ$$
$$E_{2y} = E_2 \sin 45^\circ$$

The $x$ and $y$ components of the net electric field are:

$$(E_{\text{net}})_x = E_{1x} + E_{2x} = E_1 \cos 45^\circ + E_2 \cos 45^\circ = 2500 \text{ N/C}$$

$$(E_{\text{net}})_y = E_{1y} + E_{2y} = -E_1 \sin 45^\circ + E_2 \sin 45^\circ = 0 \text{ N/C}$$

$$\Rightarrow E_{\text{net } at \text{ dot}}^2 = (2500 \text{ N/C}, \text{ along } +x\text{ axis})$$

Thus, the strength of the electric field is 2500 N/C and its direction is horizontal.
Assess: Because the charges are located symmetrically on either side of the y-axis and are of equal value, the y-components of their fields will cancel when added.

**P20.26. Prepare:** A field is the agent that exerts an electric force on a charge. Because the weight of the plastic ball acts downward, the electric force must act upward.

\[
\vec{F}_{\text{on } q} = -mg
\]

Solve: Newton’s second law on the plastic ball is \( \Sigma(\vec{F}) = \vec{a} \). \( \Sigma(\vec{F}) = \vec{w} + \vec{F}_{\text{on } q} \). To balance the weight with the electric force,

\[
F_{\text{on } q} = w \Rightarrow |q|E = mg \Rightarrow E = \frac{mg}{|q|} = \frac{(1.0 \times 10^{-3} \text{ kg})(9.8 \text{ N/kg})}{3.0 \times 10^{-9} \text{ C}} = 3.3 \times 10^6 \text{ N/C}
\]

Because \( F_{\text{on } q} \) must be upward and the charge is negative, the electric field at the location of the plastic ball must be pointing downward. Thus \( \vec{E} = (3.3 \times 10^6 \text{ N/C}, \text{ downward}) \).

Assess: \( \vec{F} = q\vec{E} \) means the sign of the charge \( q \) determines the direction of \( \vec{F} \) or \( \vec{E} \). For positive \( q \), \( \vec{E} \) and \( \vec{F} \) are pointing in the same direction. But \( \vec{E} \) and \( \vec{F} \) point in opposite directions when \( q \) is negative.

**P20.36. Prepare:** Equation 20.8 tells us the force on a charged object in an electric field: \( \vec{F}_{\text{on }} = q\vec{E} \).

We are given \( q = e \) and \( E = 1.0 \times 10^7 \text{ N/C} \).

Solve:

\[
F_{\text{on } q} = qE = (1.6 \times 10^{-19} \text{ C})(1.0 \times 10^7 \text{ N/C}) = 1.6 \times 10^{-12} \text{ N}
\]

Assess: Notice the C’s cancel out leaving units of N. The answer is very small, but that is what we expect for such a small charge.

**P20.42. Prepare:** The two charged spheres are point charges. The electric force on one charged sphere due to the other charged sphere is equal to the sphere’s mass multiplied by its acceleration. Because the spheres are identical and equally charged, \( m_1 = m_2 = m \) and \( q_1 = q_2 = q \).

Solve: We have

\[
F_{\text{on } 1} = F_{\text{on } 2} = \frac{Kq_1q_2}{r^2} = \frac{Kq^2}{r^2} = ma
\]

\[
\Rightarrow q^2 = \frac{mar^2}{K} = \frac{(1.0 \times 10^{-3} \text{ kg})(225 \text{ m/s}^2)(2.0 \times 10^{-3} \text{ m})^2}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.0 \times 10^{-14} \text{ C}^2
\]

\[
\Rightarrow q = 1.0 \times 10^{-7} \text{ C} = 100 \text{ nC}
\]

**P20.55. Prepare:** The charges are point charges. We will denote the charges \( -Q, 4Q \) and \( -Q \) by 1, 2, and 3, respectively.
It is clear that the magnitude of the force by 2 on \( q \) is larger by a factor of 2 compared to the force by 1 or by 3:

\[ F_{2\, on\, q} = 2KQq/L^2 = 2F_{1\, on\, q} = 2F_{3\, on\, q}. \]

The force vectors are shown in the previous figure.

(b)

\[ (\vec{F}_{\text{net}})_x = -\frac{KQq}{L^2} + \frac{2KQq}{L^2} \cos 45^\circ = -\frac{KQq}{L^2} (1 - \sqrt{2}) \]
\[ (\vec{F}_{\text{net}})_y = -\frac{KQq}{L^2} + \frac{2KQq}{L^2} \sin 45^\circ = -\frac{KQq}{L^2} (1 - \sqrt{2}) \]

\[ \Rightarrow F_{\text{net}} = \sqrt{\left(\frac{KQq}{L^2} (1 - \sqrt{2})\right)^2 + \left(\frac{KQq}{L^2} (1 - \sqrt{2})\right)^2} = (2 - \sqrt{2}) \frac{KQq}{L^2} \]

**Assess:** Because the \( x \)- and \( y \)-components of the net force are equal, the net force is away from +4Q connecting it to +q.
P20.58. Prepare: We have sufficient information to determine the electric force on the charged bee and the weight of the bee. Knowing these two forces we can determine their ratio. When these two forces are equal in magnitude and opposite in direction, the bee will hang suspended in air.

Solve: (a) The ratio of the electric force to the weight is determined by:

\[
\frac{F_x}{w} = \frac{qE}{mg} = 2.3 \times 10^{-6}
\]

(b) The bee will hang suspended when the electric force is equal to the weight: \(F_x = w\) or \(qE = mg\).

This gives an electric field of \(E = mg/q = 4.3 \times 10^6\) N/C.

Assess: Table 20.2 informs us that an electric field of \(10^6\) N/C will create a spark in air. Note that the required electric field is greater than the air breakdown electric field. As a result we don’t expect the bees to just hang in the air due to the charge acquired while flying; they will have provide some of the lift.

P20.59. Prepare: The field due to the third charge \((-10 \text{ nC})\) must be equal in magnitude and opposite in direction to the vector sum of the contributions to the field at the origin by the other two charges. We’ll use Equation 20.6 for the field from a point charge:

\[
E = \frac{K|q|}{r^2}
\]

with the direction away from \(q\) if \(q > 0\) and toward \(q\) if \(q < 0\).

This problem is most elegantly done if we use letters to represent the values; this will save a lot of writing, and will actually make the algebra easier. Say that \(q = 5.0 \text{ nC}\) so that the charge on the \(x\)-axis is \(-q\) and the charge on the \(y\)-axis is \(2q\). Also let \(a\) stand for the distance to the closest charge, 5.0 cm; this means the distance to the +10 nC charge is \(2a\).

Solve: The field due to the first two charges is:

\[
E_x = \frac{K|q|}{a^2} \quad \text{(due to the } -5.0 \text{ nC charge)}
\]

in the positive \(x\)-direction; and

\[
E_y = \frac{K|2q|}{(2a)^2} = \frac{K|q|}{2a^2} \quad \text{(due to the } +10 \text{ nC charge)}
\]

in the negative \(y\)-direction. You can see that \(E_x = 2E_y\). The magnitude of the total field due to the first two charges is then

\[
E = \sqrt{E_x^2 + E_y^2} = \frac{K|q|}{a^2} \sqrt{1^2 + \left(1 \frac{1}{2}\right)^2} = \frac{K|q|}{a^2} \sqrt{\frac{5}{4}} = \frac{K|q|}{a^2} \frac{\sqrt{5}}{2}
\]

Because \(E_x = 2E_y\), the direction of the total field is \(\alpha = \tan^{-1}(-E_y/E_x) = \tan^{-1}(-1/2) = -26.57^\circ\).

We now have the magnitude and direction of the total field from the first two charges. The field due to the third charge must have the same magnitude and opposite direction. The third charge has charge \(-2q\), and is \(r_3\) away from the origin. We need to solve for \(r_3\):

\[
E_3 = \frac{K|q|}{a^2} \frac{\sqrt{5}}{2} = \frac{K|-2q|}{r_3^2}
\]

This implies that
The direction of $E_3$ is $\theta = \alpha + 180^\circ = 153.43^\circ$.

We now have the distance from the origin and direction of the third charge; we can use these to get the $x$- and $y$-coordinates.

$$x = r_3 \cos \theta = \frac{2a}{\sqrt{5}} \cos 153.43^\circ = -5.98 \text{ cm} \approx -6.0 \text{ cm}$$

$$y = r_3 \sin \theta = \frac{2a}{\sqrt{5}} \sin 153.43^\circ = 2.00 \text{ cm} \approx 3.0 \text{ cm}$$

The final result is that $(x, y)$ for the $-10 \text{ nC}$ charge is $(-6.0 \text{ cm}, 3.0 \text{ cm})$.

**Assess:** A quick sketch of the coordinate plane with the three charges and their field contributions at the origin should convince you that we have the right answer.

The fact that we never need to plug in the actual value of $q$ anywhere shows that result is independent of $q$ as long as the ratios of the $q$'s are the same. Furthermore, since $K$ also canceled, then the result would hold for any inverse-square field, such as a gravitational field if you could find both positive and negative masses.

**P20.69. Prepare:** When the bead is at rest the sum of the forces on it is zero, so the magnitude of the force from the $4q$ charge at the right must be the same as the magnitude of the force from the $q$ charge at the left. This can be solved with Coulomb’s law and some algebra, but it is easier to just reason about it.

**Solve:** Since Coulomb’s law is an inverse square law, the $4q$ charge needs to be twice as far away from the free-to-move charge as the $q$ on the left is. The place that is twice as far away from $x = 4.0 \text{ cm}$ as the origin is one-third the way from the origin, or at $x = 1.3 \text{ cm}$.

**Assess:** The charge that is twice as far away has four times the charge to produce the same magnitude force.