Name: ______________________________

Test 2 Review!

1. Suppose a 90% CI is built and the interval is from (3.57, 6.46). If we wanted to then create a 95% CI, would we expect this interval to get narrower or wider?
   Wider.
2. In 1882 Michelson measured the speed of light. His values are in km/sec and have 299,000 subtracted from them. He reported the results of 23 trials with a mean of 756.22 and a standard deviation of 107.12.
   a. Find a 95% CI for the true speed of light from these statistics.
   $$n = 23, \bar{x} = 756.22, s = 107.12$$
   $$t_{\frac{\alpha}{2},n-1} = t_{.025,22} = 2.074$$
   $$(709.9, 802.5)$$
   We are 95% confident that the speed of light is between 299,709.9 $\frac{km}{sec}$ and 299,802.5 $\frac{km}{sec}$.
   b. What assumptions must you make in order to use your method?
   The population is approximately normal.
3. Let μ denote the true average radioactivity level. The value 5 pCi/L is considered the dividing line between safe and unsafe water. Would you recommend testing H0: $\mu = 5$ vs H1: $\mu > 5$ or H0: $\mu = 5$ vs H1: $\mu < 5$? Why?
   We'd like to conclude the water is significantly safe rather than significantly unsafe, so the best option for the setup of the null and alternative hypothesis would be: H0: $\mu = 5$ vs H1: $\mu < 5$.

4. A 99% CI for a population mean based on 99 observations is found to be (2.72, 3.14). How many observations must be made so that a 99% CI will specify the mean to within ± 0.1?
   First solve for s. Note that the initial ME is 0.21. We use this to solve for s.
   $$0.21 = 2.58 * \frac{s}{\sqrt{99}} => s = 0.81$$
   Then we use our newly found standard deviation to solve for the necessary sample size.
   $$0.1 = 2.58 * \frac{0.81}{\sqrt{n}} => n = 437$$

5. Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. Find the probability that at least 8 fail the test.

$$X \sim Bin(15{,}0.2)$$
$$P(X \geq 8) = 1 - P(X \leq 7) = 1 - binomcdf(15{,}0.2{,}7) = .0042$$

6. Suppose the number X of tornadoes observed in a particular region during a 1-year period has a Poisson distribution with $\lambda = 8$. What is the probability that 5 tornadoes are seen in a 6 month period?

$$\lambda = 8 \frac{occurances}{1\ year}\ or\ 4 \frac{occurances}{6\ months}$$
$$X \sim Poisson(4)$$
$$P(X = 5) = \frac{e^{-4} 4^5}{5!} = 0.156$$

7. Let X = time between two successive arrivals at the drive-up window of a local bank. If the expected wait time is 1 minute, what is the probability that the response time is at most 5 minutes?

In this case, λ is given through the mean. Since $\mu = \frac{1}{\lambda}$ and $\mu = 1$, then $\lambda = 1$.

Then: $X \sim Exp(1)$.

$$P(X \leq 5) = 1 - e^{-5} \approx 0.993$$

8. The time that it takes a driver to react to the brake lights on a decelerating vehicle is critical in helping to avoid rear-end collisions. It has been suggested that reaction time for an in-traffic response to a brake signal from the standard brake lights can be modeled with a normal distribution having mean value 1.25 seconds and a standard deviation of 0.46 seconds. What is the probability that reaction time is between 1 second and 1.75 seconds?

$$X \sim N(1.25, 0.46^2)$$
$$P(1 < X < 1.75)$$
$$P(-0.54 < Z < 1.09)$$
$$P(Z < 1.09) - P(Z < -0.54) = .5675$$

9. Extensive monitoring of a computer time-sharing system has suggested that response time to a particular editing command is normally distributed with standard deviation of 25 milliseconds. A new operating system has been installed, and we wish to estimate the true average response time for the environment. Assuming that response times are still normally distributed with $\sigma = 25$, what sample size is necessary to ensure that the resulting 95% CI has a width of at most 10?

$X \sim N(\mu, 25^2)$ Width=10, 95% CI => $z_{\alpha/2} = 1.96$

$10 = 1.96 * \frac{25}{\sqrt{n}}$ => $\sqrt{n} = 4.9$ => $n = 24.01$ so we'd sample 25.

10. A manufacturer of sprinkler systems used for fire protection in office buildings claims that the true average system-activation temperature is $130°F$. A sample of $n = 40$ systems, when tested, yields a sample average activation temperature of $131.08°F$. If the distribution of activation times is normal with standard deviation $1.5°F$, does the data contradict the manufacturer's claim?

$$H_0: \mu = 130°F$$

$$H_1: \mu \neq 130°F$$

$$n = 40, \bar{x} = 131.08°F, \sigma = 1.5°F$$

$$z = \frac{131.08 - 130}{1.5/\sqrt{40}} = 4.55$$

$$2 * P(Z > 4.55) = flippin'small$$

There is conclusive evidence to reject the assumption the mean is $130°F$.